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MOTIVE FORCES UNDER COMPLETE MIXING OF A LIQUID AT THE CONTACT STAGE

V. N. Pavlechko

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We have found the logarithmic and arithmetic mean motive forces and the corresponding numbers of transfer units N in the vapor and liquid phases under complete mixing of a liquid. The efficiency values expressed in terms of N have been derived. It has been proved that the numbers of transfer units are equal in the complex model and the mass-exchange variants using the conditions for the coupling of an ideal and a real plate of the Murphree and Hausen models. The relationship between the N values under complete mixing and without it has been established.

Mixing of a liquid on a plate has an appreciable effect on the mass-exchange efficiency. Analysis of a complex model for the counterflow [1] and the cross flow [2] shows that under complete mixing of the liquid the calculated dependences are analogous to the respective expressions for the forward flow [3]. In particular, for the latter the concentration of the highly volatile component of the liquid before the plate from [3] in view of formula (25) of [4] is determined as

$$x_n = x_{n-1} + \frac{(m+1)\left(x_{n-1} - \frac{y_{n-1}}{m}\right)E_{\text{ff}}}{\frac{L}{V} + m + \frac{L}{mV}E_{\text{ff}} - mE_{\text{ff}}} . \tag{1}$$

With the use of the material balance equation

$$L\,(x_n - x_{n-1}) = V\,(x_n - x_{n-1}) \tag{2}$$

the concentration of the highly volatile component in the vapor phase after the plate is

$$y_n = y_{n-1} + \frac{(m+1)\left(x_{n-1} - \frac{y_{n-1}}{m}\right)\frac{L}{V}E_{\text{ff}}}{\frac{L}{V} + m + \frac{L}{mV}E_{\text{ff}} - mE_{\text{ff}}} . \tag{3}$$

Upon complete mixing the composition of the liquid on the plate is equal to the composition after it and is independent of the mutual direction of the vapor and the liquid. Under these conditions, the motive force expressed in terms of the liquid parameters at the vapor inlet to the plate and at its outlet is equal to

$$\Delta x_{\text{in}} = x_{n-1} - \frac{y_{n-1}}{m} = \frac{x_n - x_{n-1}}{E_{\text{ff}}\,(m+1)}\left(\frac{L}{V} + m + \frac{L}{mV}E_{\text{ff}} - mE_{\text{ff}}\right), \tag{4}$$

$$\Delta x_{\text{f}} = x_{n-1} - \frac{y_n}{m} = \frac{x_n - x_{n-1}}{E_{\text{ff}}\,(m+1)}\left(\frac{L}{V} + m\right)(1 - E_{\text{ff}}) . \tag{5}$$

Belarusian State Technological University, 13a Sverdlov Str., Minsk 220050, Belarus; email: Paulechka@tut.by. Translated from Inzhenerno-Fizicheskii Zhurnal, Vol. 78, No. 2, pp. 118–124, March–April, 2005. Original article submitted December 10, 2003.

The logarithmic mean motive force for the parameters of the liquid and vapor phases, respectively, is

$$\Delta x_{\mathrm{m,c.m,log}} = \frac{\Delta x_{\mathrm{in}} - \Delta x_{\mathrm{f}}}{\ln \dfrac{\Delta x_{\mathrm{in}}}{\Delta x_{\mathrm{f}}}} = \frac{\dfrac{L}{mV}(x_n - x_{n-1})}{\ln \dfrac{1 + \dfrac{E_{\mathrm{ff}}}{m} - \dfrac{m+1}{\dfrac{L}{V} + m} E_{\mathrm{ff}}}{1 - E_{\mathrm{ff}}}} ,$$

$$\Delta y_{\mathrm{m,c.m,log}} = \frac{\Delta y_{\mathrm{in}} - \Delta y_{\mathrm{f}}}{\ln \dfrac{\Delta y_{\mathrm{in}}}{\Delta y_{\mathrm{f}}}} = \frac{y_n - y_{n-1}}{\ln \dfrac{1 + \dfrac{E_{\mathrm{ff}}}{m} - \dfrac{m+1}{\dfrac{L}{V} + m} E_{\mathrm{ff}}}{1 - E_{\mathrm{ff}}}} . \tag{6}$$

If the ratio between the initial and final motive forces does not exceed two, then the logarithmic mean values can be replaced by the arithmetic means. For the complex model, upon stirring the liquid this ratio is equal to the argument of the logarithms in (6):

$$\frac{1 + \dfrac{E_{\mathrm{ff}}}{m} - \dfrac{m+1}{\dfrac{L}{V} + m} E_{\mathrm{ff}}}{1 - E_{\mathrm{ff}}} \le 2 , \tag{7}$$

whence

$$E_{\mathrm{ff}} \le \frac{1}{1 + \dfrac{m+1}{m\left(1 + \dfrac{mV}{L}\right)}} . \tag{8}$$

The arithmetic mean motive forces are, respectively, equal to

$$\Delta x_{\mathrm{m,c.m,a}} = \frac{\Delta x_{\mathrm{in}} + \Delta x_{\mathrm{f}}}{2} = \left[\frac{\dfrac{L}{mV} + 1}{m+1}\left(\frac{m}{E_{\mathrm{ff}}} - \frac{m-1}{2}\right) - \frac{1}{2}\right](x_n - x_{n-1}) ,$$

$$\Delta y_{\mathrm{m,c.m,c}} = \left[\frac{1 + \dfrac{mV}{L}}{m+1}\left(\frac{m}{E_{\mathrm{ff}}} - \frac{m-1}{2}\right) - \frac{mV}{2L}\right](y_n - y_{n-1}) . \tag{9}$$

From the known expression for the numbers of transfer units, using (6) and (9), the following values have been derived:

$$N_{\mathrm{liq,c.m,log}} = \frac{mV}{L} \ln \frac{1 + \dfrac{E_{\mathrm{f}}}{m} - \dfrac{m+1}{\dfrac{L}{V} + m} E_{\mathrm{ff}}}{1 - E_{\mathrm{ff}}} , \tag{10}$$

$$N_{\mathrm{v,c.m,log}} = \ln \frac{1 + \dfrac{E_{\mathrm{ff}}}{m} - \dfrac{m+1}{\dfrac{L}{V} + m} E_{\mathrm{ff}}}{1 - E_{\mathrm{ff}}} , \tag{11}$$

$$N_{\text{liq,c.m,a}} = \frac{1}{\dfrac{\dfrac{L}{mV}+1}{m+1}\left(\dfrac{m}{E_{\text{ff}}}-\dfrac{m-1}{2}\right)-\dfrac{1}{2}}\,, \tag{12}$$

$$N_{\text{v,c.m,a}} = \frac{1}{\dfrac{1+\dfrac{mV}{L}}{m+1}\left(\dfrac{m}{E_{\text{ff}}}-\dfrac{m-1}{2}\right)-\dfrac{mV}{2L}}\,. \tag{13}$$

Comparison of formulas (6), (9)–(11) and the respective values for the forward flow of phases [5] shows that they differ. Consequently, mixing appreciably influences the motive forces even at one and the same mass-exchange efficiency.

In [6], we considered mass-exchange variants, the first of which corresponds to the coupling conditions of the ideal and real plates of the Murphree model [7–9] in analyzing the efficiency in the vapor phase and liquid and the third of which corresponds to the Hausen model [8–10]. The corresponding differences in the concentrations of the highly volatile component in the liquid before and after the plate at a forward flow are of the form

$$x_n - x_{n-1} = \frac{\left(x_{n-1}-\dfrac{y_{n-1}}{m}\right)E_{\text{ff1}}}{\dfrac{L}{mV}}\,, \tag{14}$$

$$x_n - x_{n-1} = \frac{\left(x_{n-1}-\dfrac{y_{n-1}}{m}\right)E_{\text{ff2}}}{\dfrac{L}{mV}E_{\text{ff2}}+1-E_{\text{ff2}}}\,, \tag{15}$$

$$x_n - x_{n-1} = \frac{\left(x_{n-1}-\dfrac{y_{n-1}}{m}\right)E_{\text{ff3}}}{\dfrac{L}{mV}+1-E_{\text{ff3}}}\,. \tag{16}$$

In the above sequence for the complex model, we have obtained the logarithmic mean and the arithmetic mean motive forces for the considered variants of mass exchange (Table 1).

Comparison of the data of Table 1 with the respective values of Tables 1 and 3 of [11] for the forward flow reveals their marked difference. Thus, for complete mixing of the liquid on the plate, when the difference of the concentrations of the highly volatile component in the flows and the mass exchange efficiency coincide, the motive forces differ from the respective values at ideal displacement of the liquid even in the case of forward flowing phases.

From (12)–(15) and Table 1 we derived efficiencies expressed in terms of the corresponding numbers of transfer units (Table 2).

Analytical comparison of the logarithmic mean values of N for the liquid mixing on the plate and without it presents certain mathematical difficulties. In this connection, let us consider the relation between the arithmetic means.

From [12] it is known that arithmetic mean numbers of transfer units under a forward flow are determined by the liquid-phase parameters without its mixing by the expression

$$N_{\text{liq,a}} = \frac{(m+1)\,E_{\text{ff}}}{\dfrac{L}{V}+m-\left(\dfrac{L}{mV}+1\right)\dfrac{m-1}{2}\,E_{\text{ff}}}\,, \tag{17}$$

from which we have derived the efficiency value

TABLE 1. Motive Forces under Complete Stirring of the Liquid

Quantity	Variants of mass exchange		
	1	2	3
$\frac{\Delta x_{m,c.m,log}}{x_n - x_{n-1}}$	$\frac{\frac{L}{mV}}{\ln \frac{1}{1-E_{ff1}}}$	$\frac{\frac{L}{mV}}{\ln \frac{\frac{L}{mV}E_{ff2}+1-E_{ff2}}{1-E_{ff2}}}$	$\frac{\frac{L}{mV}}{\ln \frac{\frac{L}{mV}+1-E_{ff3}}{\left(\frac{L}{mV}+1\right)(1-E_{ff3})}}$
$\frac{\Delta y_{m,c.m,log}}{y_n - y_{n-1}}$	$\frac{1}{\ln \frac{1}{1-E_{ff1}}}$	$\frac{1}{\ln \frac{\frac{L}{mV}E_{ff2}+1-E_{ff2}}{1-E_{ff2}}}$	$\frac{1}{\ln \frac{\frac{L}{mV}+1-E_{ff3}}{\left(\frac{L}{mV}+1\right)(1-E_{ff3})}}$
$\frac{\Delta x_{m,c.m,a}}{x_n - x_{n-1}}$	$\frac{L}{mV}\frac{1-\frac{E_{ff1}}{2}}{E_{ff1}}$	$\frac{1-E_{ff2}+\frac{L}{mV}\frac{E_{ff2}}{2}}{E_{ff2}}$	$\frac{1-E_{ff3}+\frac{L}{mV}\left(1-\frac{E_{ff3}}{2}\right)}{E_{ff3}}$
$\frac{\Delta y_{m,c.m,a}}{y_n - y_{n-1}}$	$\frac{1-\frac{E_{ff1}}{2}}{E_{ff1}}$	$\frac{\frac{mV}{L}(1-E_{ff2})+\frac{E_{ff2}}{2}}{E_{ff2}}$	$\frac{\frac{mV}{L}(1-E_{ff3})+1-\frac{E_{ff3}}{2}}{E_{ff3}}$
$N_{liq,c.m,log}$	$\frac{mV}{L}\ln \frac{1}{1-E_{ff1}}$	$\frac{mV}{L}\ln \frac{\frac{L}{mV}E_{ff2}+1-E_{ff2}}{1-E_{ff2}}$	$\frac{mV}{L}\ln \frac{\frac{L}{mV}+1-E_{ff3}}{\left(\frac{L}{mV}+1\right)(1-E_{ff3})}$
$N_{v,c.m,log}$	$\ln \frac{1}{1-E_{ff1}}$	$\ln \frac{\frac{L}{mV}E_{ff2}+1-E_{ff2}}{1-E_{ff2}}$	$\ln \frac{\frac{L}{mV}+1-E_{ff3}}{\left(\frac{L}{mV}+1\right)(1-E_{ff3})}$
$N_{liq,c.m,a}$	$\frac{E_{ff1}}{\frac{L}{mV}\left(1-\frac{E_{ff1}}{2}\right)}$	$\frac{E_{ff2}}{1-E_{ff2}+\frac{L}{mV}\frac{E_{ff2}}{2}}$	$\frac{E_{ff3}}{1-E_{ff3}+\frac{L}{mV}\left(1-\frac{E_{ff3}}{2}\right)}$
$N_{v,c.m,a}$	$\frac{E_{ff1}}{1-\frac{E_{ff1}}{2}}$	$\frac{E_{ff2}}{\frac{mV}{L}(1-E_{ff2})+\frac{E_{ff2}}{2}}$	$\frac{E_{ff3}}{\frac{mV}{L}(1-E_{ff3})+1-\frac{E_{ff3}}{2}}$

$$E_{ff} = \frac{1}{\frac{m+1}{\frac{L}{V}+m}\frac{1}{N_{\text{liq,à}}}+\frac{m-1}{2m}}. \tag{18}$$

A dependence analogous to (18) but pertaining to the regime of complete mixing of the liquid on the plate has been obtained from (12):

$$E_{ff} = \frac{1}{\frac{m+1}{\frac{L}{V}+m}\left(\frac{1}{N_{liq,c.m,a}}+\frac{1}{2}\right)+\frac{m-1}{2m}}. \tag{19}$$

The left-hand sides of (18) and (19) represent one and the same quantity, since the mass-exchange efficiency under a forward flow does not depend on the liquid mixing. Consequently, the right-hand sides of these formulas

TABLE 2. Dependences of E and N under Complete Mixing

Efficiency	Logarithmic means	Arithmetic means
E_{ff}	$\dfrac{\exp\left(\frac{L}{mV}N_{liq,c.m,log}\right)-1}{\exp\left(\frac{L}{mV}N_{liq,c.m,log}\right)+\dfrac{\frac{L}{mV}-m}{\frac{L}{V}+m}}$	$\dfrac{\frac{L}{V}+m}{m+\frac{L}{mV}\frac{m-1}{2}+\frac{m+1}{N_{liq,c.m,a}}}$
	$\dfrac{\exp N_{v,c.m,log}-1}{\exp N_{v,c.m,log}+\dfrac{\frac{L}{mV}-m}{\frac{L}{V}+m}}$	$\dfrac{\frac{L}{V}+m}{m+\frac{L}{mV}\frac{m-1}{2}+\frac{mV}{L}\frac{m+1}{N_{v,c.m,a}}}$
E_{ff1}	$1-\dfrac{1}{\exp\left(\frac{L}{mV}N_{liq,c.m,log}\right)}$	$\dfrac{1}{\frac{1}{2}+\frac{L}{mV}\frac{1}{N_{liq,c.m,log}}}$
	$1-\dfrac{1}{\exp N_{v,c.m,log}}$	$\dfrac{1}{\frac{1}{2}+\frac{1}{N_{v,c.m,a}}}$
E_{ff2}	$\dfrac{1}{1+\left(\frac{L}{mV}-1\right)\dfrac{1}{\exp\left(\frac{L}{mV}N_{liq,c.m,log}\right)}}$	$\dfrac{1}{1-\frac{L}{2mV}+\frac{1}{N_{liq,c.m,a}}}$
	$\dfrac{1}{1+\left(\frac{L}{mV}-1\right)\dfrac{1}{\exp N_{v,c.m,log}}}$	$\dfrac{1}{1-\frac{L}{2mV}+\frac{L}{mV}\frac{1}{N_{v,c.m,a}}}$
E_{ff3}	$\dfrac{\exp\left(\frac{L}{mV}N_{liq,c.m,log}\right)-1}{\exp\left(\frac{L}{mV}N_{liq,c.m,log}\right)-\dfrac{1}{\frac{L}{mV}+1}}$	$\dfrac{\frac{L}{mV}+1}{1+\frac{L}{2mV}+\frac{1}{N_{liq,c.m,a}}}$
	$\dfrac{\exp N_{v,c.m,log}-1}{\exp N_{v,c.m,log}-\dfrac{1}{\frac{L}{mV}+1}}$	$\dfrac{\frac{L}{mV}+1}{1+\frac{L}{2mV}+\frac{L}{mV}\frac{1}{N_{v,c.m,a}}}$

should also be equal. From this equality the relationship between the numbers of transfer units with allowance for the liquid mixing and without it is found:

$$N_{liq,c.m,à}=\frac{N_{liq,à}}{1-\frac{N_{liq,à}}{2}}\,. \tag{20}$$

The equation obtained confirms the influence of mixing on the numbers of transfer units. In the presence of the liquid mixing N should be larger than in its absence for one and the same efficiency.

In [13], we presented the dependence of the efficiencies of the above variants of mass exchange, which, in view of the complex model of [3], is given by the equation

$$\frac{\frac{L}{V}+m}{(m+1)\,E_{ff}}+\frac{\frac{L}{mV}-m}{m+1}=\frac{\frac{L}{mV}}{E_{ff1}}=\frac{L}{mV}+\frac{1}{E_{ff2}}-1=\frac{\frac{L}{mV}+1}{E_{ff3}}-1\,. \tag{21}$$

Let us express the efficiency of the complex model in terms of the efficiencies of all three variants of mass exchange from (21):

$$E_{\rm ff} = \frac{\left(\frac{L}{V} + m\right) E_{\rm ff1}}{\frac{L}{V} + \frac{L}{mV}(1 - E_{\rm ff1}) + mE_{\rm ff1}}\,, \tag{22}$$

$$E_{\rm ff} = \frac{\left(\frac{L}{V} + m\right) E_{\rm ff2}}{\left(\frac{L}{V} - 1\right) E_{\rm ff2} + m + 1}\,, \tag{23}$$

$$E_{\rm ff} = \frac{\left(\frac{L}{V} + m\right) E_{\rm ff3}}{\left(\frac{L}{mV} + 1\right)(m + 1 - E_{\rm ff3})}\,. \tag{24}$$

Substituting (22)–(24), e.g., into (11), we obtain

$$N_{\rm v,c.m,log} = \ln \frac{1}{1 - E_{\rm ff1}}\,, \tag{25}$$

$$N_{\rm v,c.m,log} = \ln \frac{\frac{L}{mV} E_{\rm ff2} + 1 - E_{\rm ff2}}{1 - E_{\rm ff2}}\,, \tag{26}$$

$$N_{\rm v,c.m,log} = \ln \frac{\frac{L}{mV} + 1 - E_{\rm ff3}}{\left(\frac{L}{mV} + 1\right)(1 - E_{\rm ff3})}\,. \tag{27}$$

Comparison of the right-hand sides of (25)–(27) with the analogous expressions from Table 1 shows their identity. Consequently, their left-hand sides are also equal:

$$N_{\rm v,c.m,log} = N_{\rm v,c.m,log1} = N_{\rm v,c.m,log2} = N_{\rm v,c.m,log3}\,. \tag{28}$$

Upon substitution of (22)–(24) into (10), equality of the numbers of transfer units in the liquid also follows:

$$N_{\rm liq,c.m,log} = N_{\rm liq,c.m,log1} = N_{\rm liq,c.m,log2} = N_{\rm liq,c.m,log3}\,. \tag{29}$$

Dependences analogous to (28) and (29) are also derived using the arithmetic means of N.

The equality of transfer units is also proved in a different way by substituting the mass-exchange efficiency from (1), (14)–(16) into the corresponding arithmetic means of the numbers of transfer units from Table 1. In all four cases, the same dependences are derived:

$$N_{\rm liq,c.m,a} = N_{\rm liq,c.m,1} = N_{\rm liq,c.m,2} = N_{\rm liq,c.m,3} = \frac{2m\,(x_n - x_{n-1})}{2mx_{n-1} - y_n - y_{n-1}}\,. \tag{30}$$

Equal relations are also obtained with the use of the material balance equation (2) and the above formulas:

$$N_{\text{v,c.m,a}} = N_{\text{v,c.m,1}} = N_{\text{v,c.m,2}} = N_{\text{v,c.m,3}} = \frac{2\,(y_n - y_{n-1})}{2mx_{n-1} - y_n - y_{n-1}}\,. \qquad (31)$$

Expressions (30) and (31) confirm the equality of N in the vapor and liquid phases for the complex model and the considered variants.

Thus, under complete mixing of the liquid on the plate the numbers of transfer units are equal for all the mass-exchange models considered, and this equality differs from the corresponding quantity under ideal displacement of the liquid.

NOTATION

E, mass-exchange efficiency; *L*, molar flow in liquid; *m*, phase-equilibrium coefficient; *N*, number of transfer units; *V*, molar flow of vapor; *x* and *y*, concentration of the highly volatile component in the liquid and vapor phases, respectively; Δ, difference of concentrations. Subscripts: a, arithmetic value; f, finite value; log, logarithmic value; liq, liquid phase; in, initial value; *n*, number of the considered plate; $n - 1$, number of the previous plate in the course of vapor motion; ff, forward flow; c.m, complete mixing of liquid; m, mean value; v, vapor phase; 1–3, variants of mass exchange.

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